RECENT DEVELOPMENTS IN HARMONIC ANALYSIS AND THEIR APPLICATIONS

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Abstracts

Characterizations of some function spaces by the discrete Radon transform on $\mathbb{Z}^n$

Ahmed Abouelaz
(University Hassan II, Casablanca)

Let $\mathbb{Z}^n$ be the lattice in $\mathbb{R}^n$ and $\mathbb{G}$ the set of all discrete hyperplanes in $\mathbb{Z}^n$. Similarly as in the Euclidean case, for a function f on $\mathbb{Z}^n$, the discrete Radon transform Rf is defined by the integral of f over hyperplanes, and R maps functions on $\mathbb{Z}^n$ to functions on $\mathbb{G}$. In this talk we determine the Radon transform images of the Schwartz space $\mathcal{S}(\mathbb{Z}^n)$, the space of compactly supported functions on $\mathbb{Z}^n$, and a discrete Hardy space $H^1(\mathbb{Z}^n)$. This is a joint work with Takeshi Kawazoe.

Uncertainty inequalities on discrete hypergroups

Miloud Assal
(King Khalid University)

We consider a finite collections of particles (particles type) $\mathbf{C} = \{c_0, c_1, ...c_n\}$, c_0 is being a photon, which are allowed to interact by colliding. When two particles collide they coalesce to form a third particle. The results of collisions are not however definite. If we collide c_i with c_j the probability of emerging with single particle c_k is $n_{i,j}^k$ is fixed, We inspire from this abstract particle collisions a hypergroup structure on $\mathbf{C}$ by letting $c_i c_j = \sum_k n_{i,j}^k c_k$.

The classical uncertainty principal stats that a function and its Fourier transform cannot both be arbitrarily concentrated, while in the setting of finite hypergroups there is no obvious generalization of the uncertainty principal above since it does not seem to make sense. We show in this paper that a more general principals holds taking the following form,

$$|\sup f||\sup \hat{f}| \geq |\mathbf{C}|. \tag{1}$$

Clearly the cardinality of the support is not the most precise measurement of the concentration of a function. One possible improvement is based on the notion of Enropy to minimize the inequality (1).

Théorème d'Atiyah homotopique pour le groupe linéaire infini

Khadija Azi
(University Hassan II, Casablanca)

Dans cet exposé nous prouvons le théorème d'Atiyah homotopique pour le groupe linéaire infini. Dans les années 60 Atiyah a défini, pour tout groupe G, un morphisme liant l'anneau des représentations complexes de G et l'anneau de la K-théorie topologique. Ce morphisme se factorise à travers le complété par rapport à l'idéal d'augmentation, en un isomorphisme si le groupe G est fini. Au début des années 70, Atiyah et Segal ont étendu ce résultat aux groupes de Lie compacts. Rector en 1974 démontre que ce morphisme d'Atiyah est un isomorphisme en prenant les représentations d'un groupe fini G dans un corps fini dont la caractéristique est première à l'ordre de G et en remplaçant l'anneau de la K-théorie topologique par celui de la K-théorie algébrique. Hamraoui-Kahn, en 2001, ont prouvé un analogue orthogonal du théorème de Rector. Enfin Riou, en 2006, prouve le théorème d'Atiyah homotopique pour GL_d.

Harmonic analysis on some homogeneous spaces of discrete type and differential operators

Ali Baklouti
(University of Sfax)

We discuss in this talk some problems related to a monomial representation $\tau = \mathrm{ind}_H^G \chi$ where G denotes an exponential solvable Lie group, H an analytic subgroup of G and χ a unitary character of H. We first record some results where G is nilpotent. Passing from nilpotent to exponential, many

complications arise. The goal is to put the emphasis on the case where the multiplicities of τ are of discrete type. The Plancherel formula and the algebra of differential operators on the homogeneous space G/H are studied. This is a joint work with H. Fujiwara and J. Ludwig.

On the approximat solutions of a Pexider Gołąb-Schinzel equation

Ahmed Charifi
(Ibn Tofail University, Kénitra)

In this talk we establish approximate solutions for the pexiderized Gołąb-Schinzel equation $f(x+yf(x)) = g(x)h(y)$ under the condition that $\lim_{t\to\infty} h(tx)$ exits. We also characterize the class of functions for which this equation is super-stable.

Conformally covariant trilinear forms and differential operators on the sphere

Jean-Louis Clerc
(Henri Poincaré University, Nancy)

Let S be the unit sphere in the Euclidean space $\mathbb{R}^n$, and let $G = SO_0(1,n)$ be the conformal group of the sphere. For $\lambda \in \mathbb{C}$, let π_λ be the principal series representation of G, acting on $\mathcal{C}^\infty(S)$. For three complex parameters $\lambda_1, \lambda_2, \lambda_3$, we construct a continuous trilinear form on $\mathcal{C}^\infty(S)\times\mathcal{C}^\infty(S)\times\mathcal{C}^\infty(S)$ which is invariant under $\pi_{\lambda_1}\otimes\pi_{\lambda_2}\otimes\pi_{\lambda_3}$. The form is obtained by meromorphic continuation in the λ's, with simple poles along four families of (explicitly defined) planes in $\mathbb{C}^3$. The trilinear form is unique (up to a factor). The residues along the planes of poles are computed. Some Bernstein-Sato identities play a major role in the proof. The residues are singular distributions, and their expression involve covariant differential operators (for three families ol planes), and covariant bidifferential operators (for the fourth family).

Further results in Dunkl analysis

Radouan Daher
(Hassan II University, Casablanca)

Making use of Dunkl's theory, we prove two estimates in the space with power weight of square integrable funcions on certain classes of functions characterized by the generalized continuity modulus. We treat some questions concerning approximation of functions in in $L2$ -metric on with power weight by functions with bounded spectrum. We prove analogs of the direct Jackson's theorem on upper bounds for the best approximations of a function in terms of its modulus of continuity and smoothness.

Some results on harmonic analysis on reductive p-adic symmetric spaces

Patrick Delorme
(Aix-Marseille University)

Let G be the group of F-points of reductive group defined over the non archimedean local field F of characteristic zero. Let H be the fixed point group of a rational involution of G.
The abstract Plancherel theorem disintegrates $L^2(G/H)$ into irreducible representations, the Dirac measure at eH into H-fixed linear forms an the functions on G/H in terms of generalized coefficients.
Our goal is to make the Placherel theorem explicit modulo the knowledge of the discrete spectrum.
For this purpose, with P. Blanc, we have built families of such forms on certain parabolically induced representations. The corresponding generalized coefficients are called Eisenstein integrals.
S.-I. Kato and K. Takano on one hand and N. Lagier on the other hand have introduced the constant term of an H-fixed linear form on a finite length smooth representation. It is a linear form on Jacques modules.
We will try to present this material together with our recent joint work with J. Carmona, which describes the constant terms of the families built with P. Blanc. This joint work with J. Carmona has important consequences on the asymptotic behavior of Eisenstein integrals. In particular it implies a temperedness result, whose proof in the real case used Oshima-Matsuki results on discrete series.

Some geometric aspects of real bounded symmetric domains

Fernando De Oliveira
(Henri Poincaré University, Nancy)

Let $\mathcal{D}_{\mathbb{C}}$ be an irreducible bounded symmetric domain in complex vector space $V_{\mathbb{C}}$. Let $V \subset V_{\mathbb{C}}$ be a real form of $V_{\mathbb{C}}$ and let $\mathcal{D} = \mathcal{D}_{\mathbb{C}} \cap V$ be the corresponding real form of $\mathcal{D}_{\mathbb{C}}$. $\mathcal{D}$ is called a real bounded symmetric domain if the involution with respect to V preserves $\mathcal{D}$. In this case $\mathcal{D}$ is Riemannian symmetric space $\mathcal{D} = G/K$ with the induced metric from $\mathcal{D}_{\mathbb{C}}$. In this talk we consider the Shilov boundary $\mathcal{S}$ of $\mathcal{D}$ (a K orbit of a maximal tripotent of V) and the diagonal action of G on $\mathcal{S} \times \mathcal{S}$. We prove that there is a finite number of orbits for which we give representative elements. The action of G on the set of transversal triples in $\mathcal{S} \times \mathcal{S} \times \mathcal{S}$ is also considered.

Family of inequalities for the Jacobi semigoup

Said Fahlaoui
(Moulay Ismail University, Meknès)

We study the Jacobi semigroup $(P_t^{\alpha,\beta})_{t\geq 0}$, $\alpha, \beta > -1$, generated by the operator $L^{\alpha,\beta} f := (1-x^2)f'' + [(\beta-\alpha) - (\alpha+\beta+2)x]f'$ acting on the Hilbert space $L^2([-1,+1], \mu_{\alpha,\beta})$ with respect to the normalized Jacobi probability measure $\mu_{\alpha,\beta}(dx) = c_{\alpha,\beta}(1-x)^{\alpha}(1+x)^{\beta}dx$. By means of a method involving essentially a commutation property between the semigroup and the derivation, we establish a family of inequalities with the Poincaré inequality as a particular case. This is a joint work with A. Bentaleb.

Asymptotic spherical analysis

Jacques Faraut
(Pierre and Marie Curie University)

We will present in a unified way results about asymptotics of spherical functions for large dimension. For that we consider the general setting of an Olshanski spherical pair, i.e. an inductive limit (G, K) of an increasing sequence of Gelfand pairs $(G(n), K(n))$,

$$G = \bigcup_{n1}^{\infty} G(n), \quad K = \bigcup_{n=1}^{\infty} K(n),$$

and study the asymptotics of a sequence (φ_n) of spherical functions for the pair $(G(n), K(n))$. Some new results have been obtained for Gelfand pairs associated to the Heisenberg group. They are related to asymptotics of multivariate Laguerre polynomials as the number of variables goes to infinity.

Complementary series for $SO(n, n+1)$

Veronique Fischer
(King's college London)

Our aim is to study the complementary series of the (real) semisimple Lie group $SO_o(n, n+1)$ associated with a maximal parabolic subgroup. We started the investigations with the non-compact picture and the analytic continuation of the Knapp-Stein intertwining operator. We have obtained that there is no complementary series if n is even whereas if n is odd we show the existence of the complementary series and we can relate them to the ones of $SO_o(n+1, n+1)$.
This talk is a joint work in progress with Prof. Genkai Zhang.

Spectral analysis of the twisted Laplacian on planar mixed automorphic forms

Allal Ghanmi
(Mohammed V University, Rabat)

We characterize the space of the so-colled planar mixed automorphic forms, with respect to an equivariant pair and a given lattice of the complex plane, as the image of the Landau automorphic forms by an appropriate transform. Related spectral properties of a twisted Laplacian acting on them are explicitly described.

On Taylor coefficients of theta function associated with Weierstrass sigma-function

Ahmed Intissar
(Mohammed V University, Rabat)

For a given lattice Γ in $(\mathbb{C}, +) = (\mathbb{R}^2, +)$, we consider the following odd entire function $\theta_W(z;\Gamma)$ on $\mathbb{C}$ given by $\theta_W(z;\Gamma) = \sum_{\gamma\in\Gamma} \gamma\chi_\Gamma(\gamma) e^{-\frac{\nu}{2}|\gamma|^2+\nu z\bar{\gamma}}$, where

we set $\nu = \nu(\Gamma) = \pi/S(\Gamma)$, $S(\Gamma)$ the area of a fundamental domain of Γ and

$$\chi_\Gamma(\gamma) = \begin{cases} +1 & \text{if } \gamma/2 \in \Gamma \\ -1 & \text{if } \gamma/2 \notin \Gamma \end{cases} .$$

In this this talk we consider the Taylor coefficients $I_k(\Gamma)$ of $\theta_W(z;\Gamma)$ at $z = 0$ and we show how the coefficients $I_k(\Gamma)$ behave as lattice modular forms of weight $2k$ when Γ is running over the set of all lattices Γ in $(\mathbb{C}, +)$.

On the Wallach set for a homogeneous bounded domain

Hideyuki Ishi
(Nagoya University)

We consider a Hilbert space of holomorphic functions whose reproducing kernel is a power of the Bergman kernel of a homogeneous, not necessarily symmetric, bounded domain. Such a Hilbert space is parametrized by the Wallach set, which we shall prove to be a union of an open interval and a discrete set. The number of the points in the discrete part is at most the rank of the homogeneous domain.
We discuss the unitary representation of the holomorphic automorphism group of the domain naturally defined on the Hilbert space, and show that the representations are not equivalent if the corresponding parameters are different. To obtain our results we use both the Siegel domain realization and the bounded realization of the Harish-Chandra type.

Sur la résolution de certaines équations fonctionnelles sur un groupe localement compact

Samir Kebbaj
(Ibn Tofail University, Kénitra)

L'objectif est de déterminer les solutions continues et bornées de la courte équation fonctionnelle de d'Alembert : $\langle \delta_x * \delta_y f\rangle + \langle \delta_x * \delta_{\hat{y}} f\rangle = 2f(x)f(y)$ où x, y appartiennent à $(X; *)$ un hypergroupe quelconque. Les solutions sont exprimées en fonctions des coefficients de représentations de dimension 2 de $(X; *)$. Ensuite, on va donner quelques applications liées à l'analyse harmonique, en retrouvant des résultats déjà connus dans la litérature, tel que les résultats de Davison sur les monoïdes et ceux de Stetkaer consernant les solutions périodiques de d'Alembert dans un groupe. Dans la deuxième partie, on va étudier l?équation fonctionnelle de Wilson : $\langle \delta_x * \delta_y f\rangle + \langle \delta_x * \delta_{\hat{y}} f\rangle = 2f(x)g(y)$.

New Formulas for the Dunkl intertwining operator

Mostafa Maslouhi
(Ibn Tofail University, Kénitra)

We establish new formulas for the Dunkl intertwining operator V_k and compute it in several interesting cases. We also give examples of regular weights k for which V_k is not positive.

Unitary representations of infinite dimensional classical groups

Stéphane Mérigon
(Erlangen University)

A unitary representation π of a (possibly infinite dimensional) Lie group G is said to be *semibounded* on the cone $W \subseteq \mathfrak{g}$ if the map defined by $s_\pi(x) := \sup \operatorname{Spec} i\mathrm{d}\pi(x) \in \mathbb{R} \cup \{\infty\}$, where $\mathrm{d}\pi$ is the derived representation, is locally bounded on W. When $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$ is a symmetric Banach-Lie algebra and W an open convex cone in $\mathfrak{q}$ invariant under the adjoint action of $H = \langle \exp \mathfrak{h} \rangle$, we show that there exists a correspondence between unitary representations of G that are semibounded on W and $*$-representations of the involutive semigroup $S = H \exp iW$. These results are meant to be applied to the study of unitary representations of infinite dimensional classical groups.

Ordinary differential equations and Kac-Moody root systems

Toshio Oshima
(University of Tokyo)

We give a new general theory of Fuchsian ordinary differential equations which understands and solves fundamental problems on the equations such as representations of their solutions, irreducibility of their monodromies, their connection coefficients, their recurrence relations etc. (http://arxiv.org/abs/1102.2792) The results can be applied to the analysis of special functions including the zonal spherical functions on the symmetric spaces by restricting them to singular lines.

Some applications of harmonic analysis in the field of directional statistics

Jean-Renaud Pycke
(University of Evry)

We introduce new statistics suited for testing uniformity of circular distributions and powerful against multimodal alternatives. This interesting property is not shared by most of the classical tests of uniformity. One of these statistcs has a simple expression in terms of the geometric mean of the sample of chord lengths. The others belong to a family indexed by a continuous parameter.
A simple heuristic justification of this features in terms of Fourier analysis allows to outline generalizations of this result to other manifolds as the unit sphere.

A Benedick's type uncertainty principle on the Heisenberg group

Peetta Kandy Ratnakumar
(Harish-Chandra Research Institute, India)

An uncertainty principle, in general terms, says that a function and its Fourier transform cannot both be controlled. One such result is Benedick's theorem, which says that a function on the Euclidean space and its Fourier transform, cannot both be supported on a set of finite measure. In this lecture, we discuss an uncertainty theorem on the Heisenberg group, which in spirit, is analogous to the Benedick's theorem. The interestig difference here is that the condition on the Heisenberg group fourier transform is given in terms of its rank. This is a joint work with Prof. E.K. Narayanan.

Invariant differential operators on a class of multiplicity free space

Hubert Rubenthaler
(University of Strasbourg)

Let $(G; V)$ be a multiplicity free space with a one dimensional quotient. Over these spaces we study various algebras of differential operators. In particular if $G' = [G; G]$ is the derived group of the reductive group G, we prove that the algebra $D(V)^{G'}$ of G'-invariant differential operators with polynomial coefficients on V , is a quotient of a Smith algebra over its center. Over $\mathbb{C}$ this class of algebras was introduced by S.P. Smith as a class of algebras similar to $\mathcal{U}(\mathfrak{sl}_2)$. This allows us to describe by generators and relations the structure of $D(V)^{G'}$.

As a corollary we obtain that various "algebras of radial com- ponents" are quotients of ordinary Smith algebras over $\mathbb{C}$. We also give the complete classificatio of the multiplicity free spaces $(G; V)$ with a one dimensional quotient, and pay particular attention to the subclass of prehomogeneous vector spaces of commutative parabolic type, for which further results are obtained.

On the regular representation of a non unimodular locally compact group

Khalil Sammad
(Ibn Tofail University, Kénitra)

In this talk we present a generalization to non-unimodular locally compact groups some basic facts that were previously only known in the unimodular case. The main results concern a Plancherel-type theorem with an inversion formula for the natural restriction of the regular representation associated with Gelfand measures. The proof of the theorem uses the notion of μ-non-degeneracy of representations to determine the support of the central disintegration measure of the natural regular representation.

Matrix-valued commuting family of differential operators associated with symmetric spaces

Nobukazu Shimeno
(Kwansei Gakui University)

I present commuting family of matrix-valued differential operators whose coefficients are given by elliptic functions. These operators are generalization of radial parts of invariant differential operators on certain homogeneous vector bundles on $Sp(2, R)/U(2)$ and $SL(3, R)/SO(3)$.

Paley-Wiener theorem(s) for real reductive Lie groups

Sofiane Souaifi
(University of Strasbourg)

In the early 80's, J. Arthur proved the Paley-Wiener theorem for real reductive Lie groups. To describe the Fourier transform of the space of compactly supported smooth functions, he uses the so-called Arthur-Campoli relations. More recently, P. Delorme, using other techniques, gave another proof of the Paley-Wiener theorem. His description of the Paley-Wiener space is now in

terms of intertwining conditions.
In a joint work with E. P. van den Ban, we make a detailed comparison between the two spaces, without using the proof or any validity of any of the associated Paley-Wiener theorems. This is done by use of the Hecke algebra.

Symplectic transfer

J. Robert Stanton
(Ohio State University)

The class os Heisenberg graded Lie algebras is a fertile field for algebra, arithmitic, geometry (differential and symplectic) and harmonic analysis. Using the specific example of binary cubics I will illustrate the variety of results and problems to be found in this area. This is joint with M. Slupinski.

Higher order Riesz transforms on Heisenberg group

Sundaram Thangavelu
(Indian Institute of Science)

In this talk we show that higher order Riesz transforms on Heisenberg groups can be characterised in terms of certain transformation properties under the action of the unitary group. We also show that they satisfy dimension-free bounds on L^p spaces under some assumptions on their multipliers. Results for reduced Heisenberg groups and multiple Laguerre expansions can be deduced by means of transference. This is joint work with P. K. Sanjay.

Harmonic Analysis and Moyal Star Products on Compact Symmetric Spaces

Harald Upmeier
(Marburg University)

In joint work with M. Englis, Prague, we consider the well-known Toeplitz-Berezin quantization for complex symmetric spaces of non-compact type (Cartan domains) and compact type (generalized Grassmann manifolds). The main goal is to develop asymptotic expansions for the associated Berezin transform and the Moyal type star products (Wick and anti-Wick quantization). Using the Peter-Weyl decomposition under the maximal compact subgroup, we find explicit formulas for the coefficients and invariant differential operators involved in the asymptotic series, which are new even for rank one spaces.

Some of the results also hold in the more general setting of real symmetric spaces.

Optimal estimates of heat kernels on nilpotent groups of Siegel type

Genkai Zhang
(Chalmers-Göteborg University)

We consider the sub-Laplacian operator on nilpotent groups equipped with the sub-Riemannian metric. There are some general known results for estimates of heat kernel of the sub-Laplacian outside the cut-locus. However estimates on the cut-locus turn out rather difficult. We prove optimal estimates for the heat kernel for the free step-two nilpotent group using some integral formula of Harish-Chandra.

Geometric properties of the magnetic Laplacian on the Euclidean 4-space

Azzouz Zinoun
(University of Lille)

When the four-dimensional Euclidean space is endowed with a covariant derivative that is either self-dual or anti-self-dual, and of constant curvature, the corresponding magnetic Laplacian is closely related to the sub-Laplacian of the quaternionic Heisenberg group. Some geometric properties of this operator are studied. In particular, it is proved that there exists a canonical orthogonal complex structure which provides a factorization in the sense of Schrödinger. We describe the main spectral properties (eigenspaces, associated reproducing kernels, heat kernel)of the magnetic Laplacian on the Euclidean four-space, regarded as acting on functions of two complex variables. Some spectral properties of the corresponding Dirac operator are also given. We give the factorization of the magnetic Laplacian in terms of differential operators on functions of one quaternionic variable.

Organizers : Boussejra Abdelhamid (Ibn Tofial University) ; Kawazoe Takeshi (Keio University) ; Koufany Khalid (Henri Poincaré University)

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